Primordial magnetic fields

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Outline

- Why do we care about primordial magnetic fields?
- Constraints on magnetic fields in the early Universe
  - From direct observations
  - From CMB
  - From nucleosynthesis
- Primordial seed magnetic fields
  - From inflation
  - From cosmological phase transitions
  - At recombination
  - From density perturbations

Outline

- Electroweak physics and primordial fields
  - Magnetic fields versus hypermagnetic fields
  - Baryon asymmetry and hypermagnetic fields
  - Magnetic helicity from electroweak anomaly
- Conclusions

Why do we care about primordial magnetic fields?

Origin of galactic and cluster magnetic field?

Probably, amplification of seed magnetic fields via galactic dynamo.

Astrophysic origin of seeds: Biermann battery in intergalactic shocks, stellar magnetic fields, supernova explosions, galactic outflows into intergalactic medium, quasar outflow of magnetized plasma,...

Cosmology: Perhaps, seeds are coming from some peculiar processes in the early universe (primordial magnetic fields $\equiv$ fields prior to recombination)?

Magnetized plasma at early times?

Change of a standard view on the evolution of the universe: inflation, cosmological phase transitions, baryogenesis, nucleosynthesis...

Constraints on magnetic fields

Notations: ρ_c - critical density, L - comoving distance,

$\Omega_B = \frac{\frac{1}{2}B^2}{\rho_c}$, $\Omega_\gamma = \frac{\frac{\pi^2}{15}T^4}{\rho_c}$

From direct observations

- Galactic magnetic fields:

  $B \sim 10^{-6}$ Gauss, at $L \sim 10$ kps, $\frac{\Omega_B}{\Omega_\gamma} \sim O(10^{-1})$

- Cluster magnetic fields

  $B \sim 10^{-7} - 10^{-6}$ Gauss, coherence scale $L \sim 10$ (?) kps,

  $\frac{\Omega_B}{\Omega_\gamma} \sim O(10^{-1}) - O(10^{-3})$

- Intergalactic magnetic fields, QSO + Faraday rotation (Kronberg; Blasi, Burles, Olinto)

  $B < 10^{-8} - 10^{-11}$ Gauss at $L \sim 1 - 50$ Mps,

  $\frac{\Omega_B}{\Omega_\gamma} < O(10^{-5}) - O(10^{-11})$

Constraints on magnetic fields

From CMB

Caprini, Durrer, Kahniashvili, Mack, Kosowsky, Pogosian, Vachaspati, Winitzki, Jedamzik, Battaner talk...

Stochastic magnetic field:

$$\langle B_i^*(k) B_j(k) \rangle \sim \left(\delta_{ij} - \hat{k}_i \hat{k}_j \right) P_B(k) + i \epsilon_{ijl} \hat{k}_l P_H(k)$$

P_B - symmetric power spectrum: $P_B \propto \langle |\vec{B}^2| \rangle$,

P_H - helical power spectrum $P_H(k) \propto \vec{B} \cdot (\vec{\nabla} \times \vec{B})$

Nonzero magnetic field: anisotropic stress-energy tensor, gravitational waves, vorticity

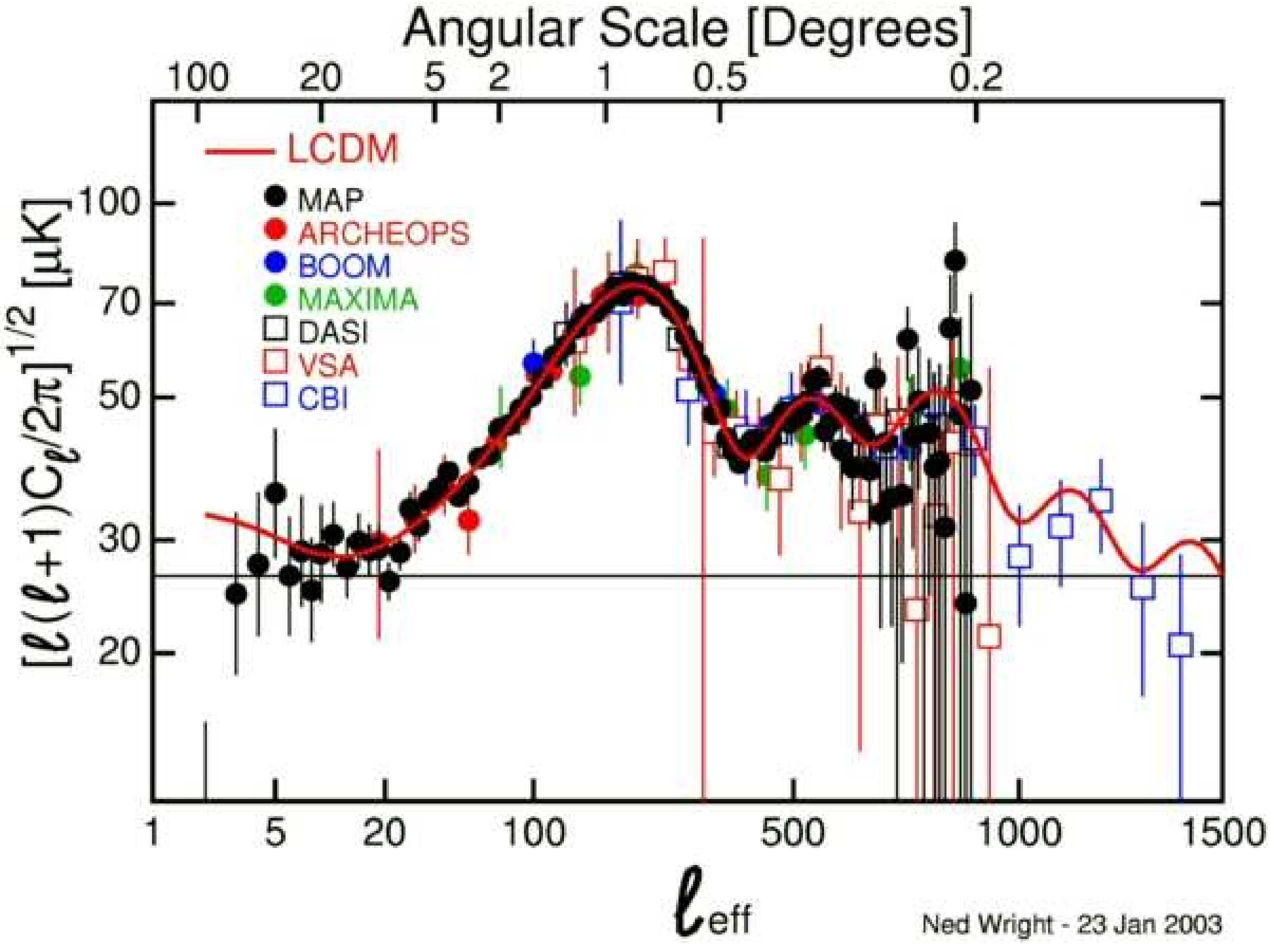
Constraints on magnetic fields: CMB

Symmetric part P_B: contributes to temperature and polarization anisotropies via induced vector and tensor perturbations

Helical part P_H: parity odd CMB fluctuations, temperature-polarization correlations

Homogeneous magnetic field: breaking of spatial isotropy, non-Gaussian CMB anisotropies, correlations between l and $l \pm 2$ multipole modes

Constraints on magnetic fields: CMB



$B < 10^{-8} - 10^{-9}$ Gauss at $L > 50$ Mps,

$\frac{\Omega_B}{\Omega_\gamma} < O(10^{-5}) - O(10^{-7})$

Constraints on magnetic fields

From nucleosynthesis

Main effect: Change of the expansion rate of the universe: change in ^4He abundance.

Other effects: Change in reaction rates, e.g. for $B > B_c = \frac{e}{m_e^2} \sim 4 \times 10^{13}$ Gauss β decay rate on neutrons is significantly increased

Greenstein; Grasso, Rubinstein:

$B(T = 10^9 K) < 10^{11}$ Gauss,

$\frac{\Omega_B}{\Omega_\gamma} < 0.05$

Constraints on magnetic fields

Small scale magnetic fields.

Let the typical scale of primordial magnetic field L is $L = l_0/T$.

Conservative survival estimate estimate: linear regime, no turbulence:

$$\frac{\partial B}{\partial t} + 2HB = -\frac{1}{\sigma}\vec{\nabla}^2 B$$

H - Hubble constant, σ - plasma conductivity:

$$\sigma \simeq \begin{cases} (0.76 - 6.7)T, & 1\ \text{MeV} < T < 100\ \text{GeV} \\ \frac{m_e}{\alpha \log(\Lambda)} \left(\frac{2T}{\pi m_e}\right)^{3/2}, & T < m_e \end{cases}$$

Constraints on magnetic fields

Small scale magnetic fields are damped away!

$$\frac{B(T)}{T^2} \propto \exp\left[-\frac{M_{Pl}}{l_0^2\sigma(T)}\right]$$

Nucleosynthesis: Only fields with $l_0 > 10^{11}$ may survive till BBN

Recombination: Only fields with $l_0 > 10^{15}$ may survive (present scale ~ 10 Au $\sim 10^{14}$ cm)

Nonlinear, MHD treatment - even stronger constraints (Banergee, Jedamzik).

No any bounds exist on small scales ($l_0 < 10^{11}$, comoving scale $< 10^9$ cm) magnetic fields! Magnetized plasma in the early universe?

Primordial seeds

Required seeds at $L \sim 1$ Mps:

Galactic dynamo: $B \sim B_0 e^{\Gamma t}$

Flat Universe: Kronberg, Beck, Brandenburg, Moss, Shukurov, Sokoloff,....

$B_{seed} \sim 10^{-19} - 10^{-23}$ Gauss,

$\frac{\Omega_B}{\Omega_\gamma} \sim O(10^{-27}) - O(10^{-35})$

Non-zero cosmological constant: Davis, Lilley, Tornkvist

$B_{seed} \sim 10^{-30}$ Gauss,

$\frac{\Omega_B}{\Omega_\gamma} \sim O(10^{-49})$

Galaxy clusters?

Even these small fields is extremely difficult to get!

MHD equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) &= 0 \\
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} &= -\frac{1}{\rho} \vec{\nabla} p - \frac{\vec{B} \times (\vec{\nabla} \times \vec{B})}{\rho} - \vec{\nabla} \Phi \\
\frac{\partial \vec{B}}{\partial t} &= \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma} \nabla^2 \vec{B} \\
\nabla^2 \Phi &= \rho
\end{aligned}
$$

(Newtonian non-relativistic limit) are linear in magnetic field!

Primordial seeds

From inflation

Quantum fluctuations of inflaton produce large scale density perturbations with $\delta\rho/\rho \sim 10^{-5}$.

Quantum fluctuations of electromagnetic field produce large scale magnetic fields?

First approximation:

$$L = -\frac{1}{4}\sqrt{-g}F^{\mu\nu}F_{\mu\nu}$$

in time dependent metric. Conformal coupling: time dependence can be removed : no particle production.

Primordial seeds

Second approximation:

$$L = \sqrt{-g}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi + ...\right]$$

EM fields couple to other fields that may couple to gravity in conformally non-invariant way → production of magnetic fields? Turner, Widrow.

- Production of charged scalars: superhorison charge and current fluctuations
- Creation of magnetic fields via A_μ-current coupling

Primordial seeds

Simplest model: scalar electrodynamics, scalar mass m_H

Giovannini, M.S. Account for : particle production in De-Sitter space, charge and current correlations, plasma conductivity

$$\frac{\Omega_B}{\Omega_\gamma} \sim \left(\frac{M_{Pl}}{m_H}\right)^3 \left(\frac{H}{M_{Pl}}\right)^5 \frac{1}{(LT)^4}$$

Numerics: $m_H \sim 100$ GeV, Hubble constant at inflation $H/M_{Pl} < 10^{-6}$, for galaxy $LT \sim 3 \times 10^{25}$,

$$\frac{\Omega_B}{\Omega_\gamma} \sim 10^{-78}$$

too small!

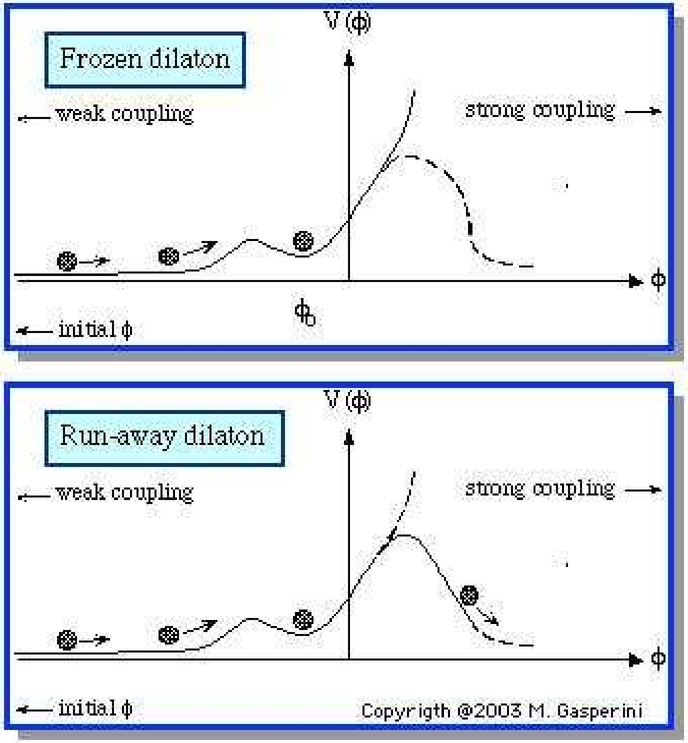
Calzetta, Kandus: Qualitatively similar estimate incorporating physics of preheating after inflation.

Primordial seeds

Dynamical gauge coupling, string cosmology

Ratra;Carroll, Field, Jackiw; Gasperini, Giovannini, Veneziano; D. and M. Lemoine,...

Idea: EM gauge coupling is dynamical, $g \sim e^{\phi}$, where ϕ is some scalar field (dilaton). Potential for ϕ:



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Primordial seeds

Time evolution of ϕ $\longrightarrow$ change of gauge coupling $\longrightarrow$ generation of magnetic fields.

The result is model dependent, but it is possible to get up to

$$\frac{\Omega_B}{\Omega_\gamma} \sim 10^{-8}$$

Primordial seeds

From cosmological phase transitions

Primordial seeds

From cosmological phase transitions

Ask Hector De Vega!

Primordial seeds

Before recombination

Typical magnetic dynamo equation: Self-excitation of magnetic fields with scale $L = \frac{\eta}{\alpha}$ with growth rate $\alpha^2/(4\eta)$,

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \alpha \vec{B} + \eta \nabla^2 \vec{B}$$

Dynamo before recombination?

Semikoz, Sokoloff: Electroweak effects of parity breaking + neutrino asymmetry $\rightarrow \alpha \neq 0$ from particle physics.

Primordial seeds

From density perturbations

Harrison

Vorticity of the plasma at $T_{rec} < T < m_e$ + mass difference of protons and electrons → non-vanishing currents → magnetic fields

Berezhiani, Dolgov;Matarrese, Mollerach, Notari, Riotto:

Nonlinear evolution of scalar density perturbations → vorticity of the plasma.

$B_{seed} \sim 10^{-29} \left(\frac{L}{Mpc}\right)^2$ Gauss,

$\frac{\Omega_B}{\Omega_\gamma} \sim O(10^{-47})$

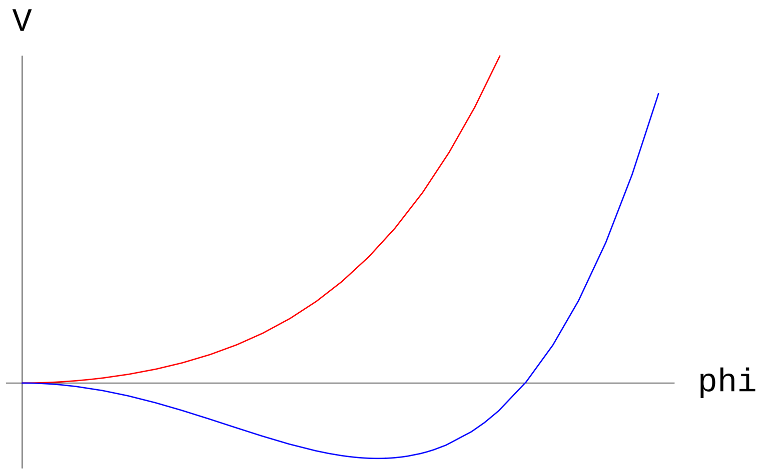
B+D: much bigger numbers

Electroweak physics and primordial fields

Magnetic fields versus hypermagnetic fields

Weak and electromagnetic forces are unified in electroweak theory

At high temperatures $T > T_c \sim 100$ GeV SU(2)xU(1) is restored: we have hypercharge magnetic field B_Y. At $T = T_c$ hypercharge field is converted in magnetic field, $B = B_Y \cos\theta_W$.



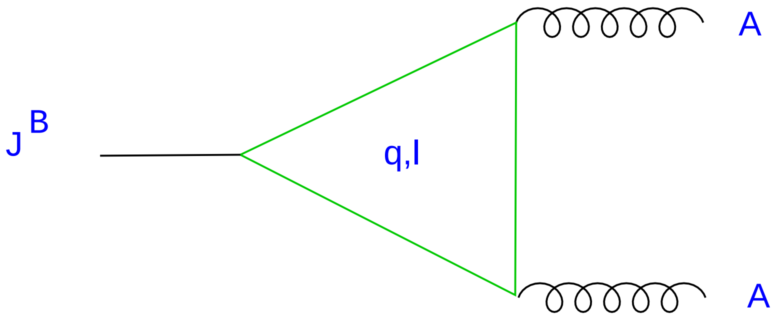
Magnetic fields versus hypermagnetic fields

Important difference: coupling of electromagnetic field to fermions is vectorlike, $\bar{\psi}\gamma_\mu\psi A^\mu$

Coupling of hypercharge field to fermions is chiral: $\bar{\psi}\gamma_\mu(a + b\gamma_5)\psi Y^\mu$

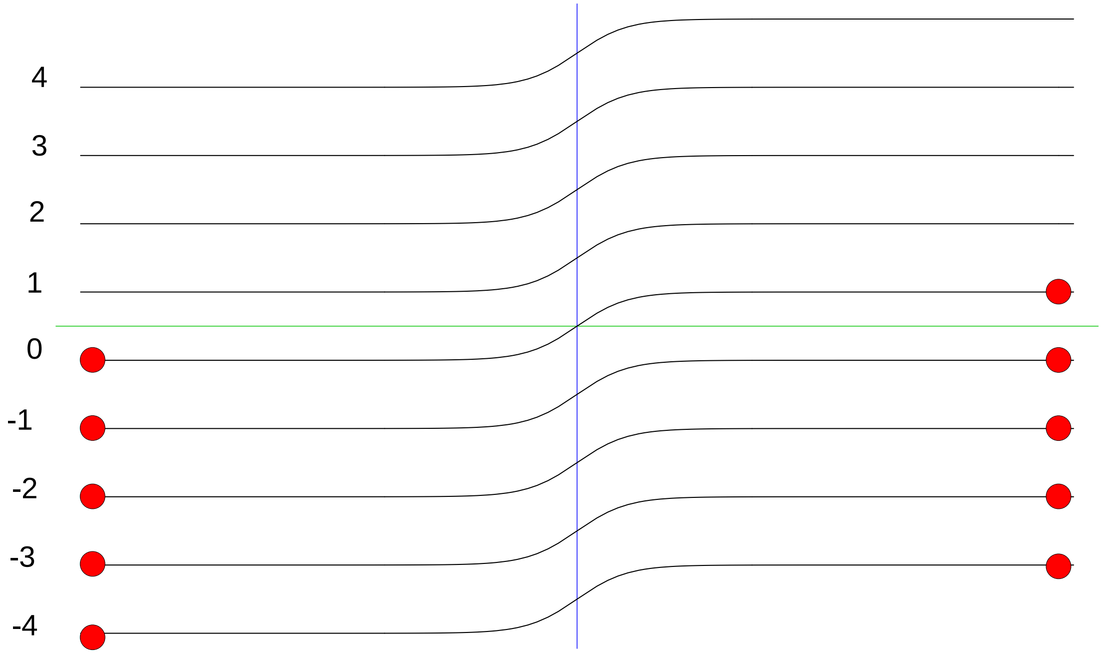
On the “classical level", baryon and lepton numbers are conserved in the standard electroweak theory. Quantum anomaly:

$$\partial_\mu J^B_\mu = \partial_\mu J^L_\mu = \frac{n_f}{32\pi^2}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right) + \frac{g'^2}{64\pi^2}\sum y_R^2\left(f_{\mu\nu}\tilde{f}_{\mu\nu}\right)$$



Electroweak B nonconservation

Physical sense: Fermionic level crossing in external hypermagnetic field



Dirac vacuum: all fermionic energy levels are occupied

One fermion is created

Selection rules

There are 12 fermionic doublets:

$(\nu_e, e),\ (\nu_\mu, \mu),\ (\nu_\tau, \tau)$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

and 21 singlets:

$e,\ \mu,\ \tau$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

$(u\ \ , d),\ (c\ \ , s),\ (t\ \ , b)$

The process with

$$\int d^4x \frac{g'^2}{64\pi^2} \frac{1}{9} \left(f_{\mu\nu} \tilde{f}_{\mu\nu} \right) = 1$$

produces 216 fermions, 72 from each generation:

$3u_L + 3d_L + 48u_R + 12d_R$

$+\nu_L + e_L + 4e_R$

Electroweak physics and primordial fields

Helicity and anomaly

Hypermagnetic helicity $\vec{Y} \cdot B_Y$= Chern-Simons number,

$\frac{\partial}{\partial t}\vec{Y} \cdot B_Y \sim f_{\mu\nu}\tilde{f}_{\mu\nu} \rightarrow$

Giovannini, MS; Joyce, MS

- Production of hypermagnetic helicity → production of baryon asymmetry
- Existence of fermionic asymmetries → generation of hypermagnetic fields

Baryon asymmetry and hypermagnetic fields

Nonzero uniform hypermagnetic helicity: production of baryon asymmetry at electroweak scale:

$$\frac{n_B}{S} \sim \frac{\alpha}{\sigma} \frac{\vec{B}_Y \cdot (\vec{\nabla} \times \vec{B}_Y)}{T_W^4} \frac{M_{Pl}}{T_W}$$

Hypermagnetic fields that can survive till EW scale:

$l_0/T, l_0 > \frac{M_{Pl}}{T_W} \sim 10^8$.

Estimate:

$$\frac{n_B}{S} \sim \frac{10^{16}}{l_0} \frac{\Omega_B}{\Omega_\gamma}$$

Correct order of magnitude for $\frac{\Omega_B}{\Omega_\gamma} \sim 10^{-18}$

Origin of hypermagnetic helicity?

Baryon asymmetry and hypermagnetic fields

Non-uniform hypermagnetic helicity fluctuations: Matter-antimatter domains.

- Astronomical scales: excluded by observations
- Scale smaller that neutron diffusion length λ_n at nucleosynthesis: dissappear without any trace
- Scale larger than λ_n: may modify predictions of standard nucleosynthesis

Giovannini, MS: Depending on the spectrum of hypermagnetic field fluctuations the constraints could be superior of that coming from BBN and CMB

Magnetic helicity from electroweak anomaly

Anomalous hypermagnetohydrodynamics

New term in equation for hypermagnetic field:

$$\frac{\partial \vec{B}_Y}{\partial t} = -\frac{4\alpha'}{\pi\sigma}\vec{\nabla} \times (\mu B_Y) + \vec{\nabla} \times (\vec{v} \times \vec{B}_Y) + \frac{1}{\sigma}\nabla^2 \vec{B}_Y$$

μ-chemical potential for anomalous charge. Looks like dynamo term!

Minimal standard model: μ is a potential for right handed electrons (can be generated in a baryogenesis process)

$\frac{\Omega_B}{\Omega_\gamma} \sim 1$ for $l_0 \sim 10^8$ at the electroweak scale.

Conclusions

- No constraints exist on the existence of small scale primordial magnetic fields
- No large scale magnetic fields are generated in inflation in the standard model
- Several mechanisms exist for creation of seed magnetic fields from particle physics
- Connection between hyper-magnetic helicity and baryon number - possible origin of the baryon asymmetry of the universe and of the galactic magnetic fields